SYLLABUS FOR MATHEMATICS

PART - I

1. REAL ANALYSIS:

Real number system, finite, countable and uncountable sets. Complete ordered field, Archimedean property, supremum, infimum. Metric spaces, compactness, connectedness.Bolzano-Weierstrass theorem, Heine-Borel theorem.Sequences and series, convergence and divergence, limsup, liminf. Limit of a function, continuity, uniform continuity, differentiability, Mean value theorems. Sequence and series of functions-uniform convergence.Riemann integrals, improper integrals.Functions of several variables-directional derivatives, Partial derivative, derivatives as a linear transformations, inverse and implicit function theorems.

2. ALGEBRA:

Groups, subgroups, normal subgroups, quotient groups, homomorphism ,cyclic groups,permutationgroups,symmetricgroups,Cayley's theorem, Sylow theorems.

Rings: ideals, prime and maximal ideals, quotient rings, Unique factorization domain, polynomial rings.

Fields: Finite fields, field extensions.

3. LINEARALGEBRA:

Vector spaces, subspaces, linear dependence, basis, dimension, algebra of linear transformations.Algebra of matrices, rank and determinant of matrices, linear equations. Eigen values and eigen vectors, Cayley-Hamilton theorem. Matrix representation of linear transformations, change of basis, Inner product spaces, orthonormal basis.

4. TOPOLOGY:

Elements of topological spaces, basis, dense sets, subspace and product topology.Compactness, Connectedness, separations axioms, first and second countability.

5. FUNCTIONALANALYSIS:

Banach spaces: Hahn-Banach theorem, open mapping and closed graph theorems. Principal of uniform boundedness, boundedness and continuity of linear transformations, dual space, embedding in the second dual, Hilbert spaces, Projections.

Orthonormal basis, Riesz-representation theorem.

PART II

1. ORDINARY DIFFERENTIAL EQUATIONS(ODEs):

Existence and Uniqueness of solutions of initial value problems for first order ordinary differential equations, singular solutions of first order ODEs, system of first order ODEs. General solutions of homogeneous and non-homogeneous linear ODEs, variation of parameters, Sturm-Liouville boundary value problem.

2. PARTIAL DIFFERENTIAL EQUATIONS(PDEs):

Lagrange method, Charpit method and Cauchy's method for solving first order PDEs. Classification of second order PDEs, General solution of higher order PDEs with constant coefficients, method of separation of variables. Laplace, Heat and wave equations.

3. COMPLEXANALYSIS:

Algebra of complex numbers, the complex plane, polar forms, Topology of complex numbers.

Analytic functions, Cauchy-Riemann equations, Harmonic functions, Conformal mappings, Mobius transformations.

Complex integrations, Cauchy's theorem's, Cauchy's integral formula, zeros and poles of analytic functions, classifications of singularities, Maximum modulus principle, Schwarz Lemma.

Calculus of residues-Cauchy's residue theorem, Argument principle, Rouch's theorem.

4. NUMERICAL ANALYSIS:

Solution of system of linear algebraic equations using matrix-inversion method, Gauss-elimination method, Gauss-Seidel method. Numerical solutions of non-linear equations. iteration method, Newton-Raphson method, rate of convergence.

Finite differences, Lagrange, Hermite and Spline interpolation.

Numerical differentiation and integration. Numerical solutions of ODEs using Picard, Euler, modified Euler and Runge-Kutta methods.

5. MEASURE THEORY:

Lebesgue measure: measurable sets, algebra of measurable sets. The class of measurable sets as an algebra, Measurable functions. Lebesgue integral, properties of Lebesgue integrals. The bounded convergence theorem, Fatou's lemma: Monotone convergence theorem; Lebesgue convergence theorem.