Syallabus for Screening test for the post of Lecturers of various subjects.

For The post of Lecturers difficulty Level of the Questions will be of Post Graduation Level.

Math

Analysis: Elementary set theory, Sets:

Sets and their representations. Empty set, Finite & Infinite sets, Equal sets. Subsets, Subsets of the set of real numbers especially intervals (with notations). Power set. Universal set. Venn diagrams. Union and Intersection of sets. Difference of sets. Complement of a set, Properties of complement sets.

Finite, countable and uncountable sets, Real number system as a complete ordered field, Archimedean property, supremum, infimum.

Sequence and series:

Sequence and Series, Arithmetic Progression (A.P), Arithmetic Mean (A.M), Geometric Progression (G.P), general term of a G.P, sum of n terms of a G.P . Arithmetic and Geometric series, infinite G.P. and its sum. Geometric mean (G .M), relation between A.M and G.M, Sum to n term of the special series $\sum n, \sum n^2$ and $\sum n^3$. Convergence,. Sequences and series of functions, uniform convergence.

Binomial Theorem:

History, statement and proof of the binomial theorem for positive integral indices. Pascal's triangle, general and middle term in binomial expansion, simple applications. Bolzano Weierstrass theorem, Heine Borel theorem.

Continuity and Differentiability:
Continuity, uniform continuity. Continuity and Differentiability, derivative of composite functions, chain rule, derivative of inverse trigonometric functions, derivative of implicit function.

Concepts of exponential and logarithmic functions. Derivatives of $\log_e^x$ and e^x. Logarithmic differentiation. Derivative of functions expressed in parametric forms.

Second order derivatives. Rolle's and Lagrange's Mean Value Theorems (without proof) and their geometric interpretations.

Applications of Derivatives:

Applications of derivatives: rate of change, increasing/decreasing functions, tangents and normal, approximation, maxima and minima. Simple problems (that illustrate basic principles and understanding of the subject as well as real life situations).

Limits and Derivatives:

Derivative introduced as rate of change both as that of distance function and geometrically, intuitive idea of limit. , limsup, liminf . Definition of derivative, relate it to slope of tangent of the curve, derivative of sum, difference, product and quotient of functions. Derivatives of polynomial and trigonometric functions. Mean value theorem.

Integrals: Integration as inverse process of differentiation. Integration of a variety of functions by

substitution, by partial fractions and by parts. Definite integrals as a limit of a sum. Fundamental Theorem of Calculus, Basic properties of definite integrals and evaluation of definite integrals. .

Applications of the Integrals:

Applications in finding the area under simple curves, especially lines, areas of circles/parabolas/ellipses, area between the two above said curves. Riemann sums and Riemann integral, Improper Integrals.

Monotonic functions, types of discontinuity, functions of bounded variation, Lebesgue measure, Lebesgue integral. Functions of several variables, directional derivative, partial derivative, derivative as a linear transformation, inverse and implicit function theorems. Metric spaces, compactness, connectedness. Normed linear Spaces. Spaces of continuous functions as examples.

Linear Algebra:
Vector spaces: Vectors and scalars, magnitude and direction of a vector. Direction cosines/ratios of vectors. Types of vectors (equal, unit, zero, parallel and collinear vectors), position vector of a point, negative of a vector, components of a vector, addition of vectors, multiplication of a vector by a scalar, position vector of a point dividing a line segment in a given ratio. Scalar (dot) product of vectors, projection of a vector on a line. Vector (cross) product of vectors, scalar triple product.

Linear Inequalities:

Linear inequalities, Algebraic solutions of linear inequalities in one variable and their representation on the number line. Graphical solution of linear inequalities in two variables. Solution of system of linear inequalities in two variables - graphically.

Subspaces, linear dependence, basis, dimension, algebra of linear transformations.

Matrices & Determinant:
Concept, notation, order, equality, types of matrices, zero matrix, transpose of a matrix, symmetric and skew symmetric matrices. Addition, multiplication and scalar multiplication of matrices, simple properties of addition, multiplication and scalar multiplication. Non-commutativity of multiplication of matrices and existence of non-zero matrices whose product is the zero matrix (restrict to square matrices of order 2). Concept of elementary row and column operations. Invertible matrices and proof of the uniqueness of inverse, if it exists.

Determinant of a square matrix (up to 3⊠3matrices), properties of determinants, minors, cofactors and applications of determinants in finding the area of a triangle.

Adjoint and inverse of a square matrix. Consistency, inconsistency and number of solutions of system of linear equation by examples, solving system of linear equations in two or three variables using inverse of a matrix. Rank and determinant of matrices, linear equations. Eigen values and eigen vectors, Cayley-Hamilton theorem.

Matrix representation of linear transformations. Change of basis, canonical forms, diagonal forms, triangular forms, Jordan forms. Inner product spaces, orthonormal basis. Quadratic forms, reduction and classification of quadratic forms
Straight Lines :

Brief recall of 2-D from earlier classes, Shifting of origin. Slope of a line and angle between two lines .Various forms of equations of a line: parallel to axes, point–slope form, slop–intercept form, two–point form, intercept form and normal form, General equation of a line. Equation of family of lines passing

through the point of intersection of two lines. Distance of a point from a line.

Conic Sections:
Sections of a cone ; circles , ellipse , parabola, hyperbola ,a point , a straight line and a pair of intersecting lines as a degenerated case of a conic section. Standard equations and simple properties of parabola, ellipse and hyperbola. Standard equations of a circle.

Introduction to Three-dimensional Geometry:
Coordinate axes and coordinate planes in three dimensions. Coordinates of a point. Distance between two points and section formula.

Three-dimensional Geometry:
Direction cosines/ratios of a line joining two points. Cartesian and vector equation of a line, coplanar and skew lines, shortest distance between two lines. Cartesian and vector equation of a plane. Angle between (i) two lines, (ii) two planes, (iii) a line and a plane. Distance of a point from a plane.

UNIT – 2
Complex Numbers and Quadratic Equations:

Need for complex numbers, especially $\sqrt{-1}$, to be motivated by inability to solve every quadratic equation. Brief description of algebraic properties of complex numbers. Argand plane and polar representation of complex numbers. Statement of Fundamental Theorem of Algebra, solution of quadratic equations in the complex number system. Square-root of a Complex number.

Algebra of complex numbers, the complex plane, polynomials, power series, transcendental functions such as exponential, trigonometric and hyperbolic functions.

Trigonometric Functions:

Positive and negative angles. Measuring angles in radians and in degrees and conversion from one measure to another. Definition of trigonometric functions with the help of unit circle. Truth of the identity $\sin^2 x + \cos^2 x = 1$, for all x. Signs of trigonometric functions and sketch of their graphs. Expressing $\sin(x \pm y)$ and $\cos(x \pm y)$ in terms of sinx, siny, cosx & cosy.

Identities related to sin 2x, cos 2x, tan 2x, sin 3x, cos 3x and tan 3x. General solution of trigonometric equations of the type $\sin\theta = \sin\alpha$, $\cos\theta = \cos\alpha$ and $\tan\theta = \tan\alpha$.

Analytic functions, Cauchy-Riemann equations. Contour integral, Cauchy's theorem, Cauchy's integral formula, Liouville's theorem, Maximum modulus principle, Schwarz lemma, Open mapping theorem. Taylor series, Laurent series, calculus of residues. Conformal mappings, Mobius transformations.

Algebra:

Permutations & Combinations:
Fundamental principle of counting, Factorial n(n!) Permutations and combinations, derivation of formulae and their connections, simple applications. Pigeon-hole principle, inclusion-exclusion principle.

Fundamental theorem of arithmetic, divisibility in Z, congruences, Chinese Remainder Theorem, Euler's $\emptyset$-function, primitive roots.

Groups, subgroups, normal subgroups, quotient groups, homomorphisms, cyclic groups, permutation

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groups, Cayley's theorem, class equations, Sylow theorems.

Rings, ideals, prime and maximal ideals, quotient rings, unique factorization domain, principal ideal domain, Euclidean domain.

Polynomial rings and irreducibility criteria. Fields, finite fields, field extensions, Galois Theory. Dense sets, subspace and product topology, separation axioms, connectedness and compactness.

UNIT – 3
Ordinary Differential Equations (ODEs):

Differential Equations:

Definition, order and degree, general and particular solutions of a differential equation. Formation of differential equation whose general solution is given.

Solution of differential equations by method of separation of variables, homogeneous differential equations of first order and first degree. Solutions of linear differential equation of the type:

$$\frac{dy}{dx} + py = Q$$

where P and Q are functions of x or constant.

$$\frac{dx}{dy} + px = Q$$

where P and Q are functions of y or constant

Existence and uniqueness of solutions of initial value problems for first order ordinary differential equations, singular solutions of first order ODEs, system of first order ODEs.

General theory of homogenous and non-homogeneous linear ODEs, variation of parameters, Sturm-Liouville boundary value problem, Green's function.

Partial Differential Equations (PDEs):
Lagrange and Charpit methods for solving first order PDEs, Cauchy problem for first order PDEs.
Classification of second order PDEs, General solution of higher order PDEs with constant coefficients, Method of separation of variables for Laplace, Heat and Wave equations.

Numerical Analysis:
Numerical solutions of algebraic equations, Method of iteration and Newton-Raphson method, Rate of convergence, Solution of systems of linear algebraic equations using Gauss elimination and Gauss-Seidel methods, Finite differences, Lagrange, Hermite and spline interpolation, Numerical differentiation and integration, Numerical solutions of ODEs using Picard, Euler, modified Euler and Runge-Kutta methods.

Calculus of Variations:
Variation of a functional, Euler-Lagrange equation, Necessary and sufficient conditions for extrema. Variational methods for boundary value problems in ordinary and partial differential equations.

Linear Integral Equations:
Linear integral equation of the first and second kind of Fredholm and Volterra type, Solutions with separable kernels. Characteristic numbers and eigenfunctions, resolvent kernel.

Classical Mechanics:

Generalized coordinates, Lagrange's equations, Hamilton's canonical equations, Hamilton's principle and principle of least action, Two-dimensional motion of rigid bodies, Euler's dynamical equations for the motion of a rigid body about an axis, theory of small oscillations.

UNIT – 4

Descriptive statistics, exploratory data analysis
Mathematical Reasoning:
Mathematically acceptable statements. Connecting words/phrases–consolidating the understanding of "if and only if (necessary and sufficient) condition", " implies", "and/or", "implied by", "and", "or", there exists" and their use through variety of examples related to real life and Mathematics , Validating the statements involving the connecting words- difference between contradiction, converse and contrapositive.

Statistics:
Measure of dispersion: mean deviation, variance and standard deviation of ungrouped/grouped data. Analysis of frequency distributions with equal means but different variances.

Probability:
Random experiments: outcomes, sample spaces(set representation). Events: Occurrence of events, 'not', 'and' & 'or' events, exhaustive events, mutually exclusive events. Axiomatic (set theoretic) probability, connections with the theories of earlier classes.

Probability of an event, probability of 'not', 'and' & 'or' events. Multiplication theorem on probability. Conditional probability, independent events, total probability, Baye's theorem, Random variable and its probability distribution, mean and variance of haphazard variable. Repeated independent (Bernoulli) trials and Binomial distribution. Sample space, discrete probability, independent events, Bayes theorem. Random variables and distribution functions (univariate and multivariate); expectation and moments. Independent random variables, marginal and conditional distributions. Characteristic functions. Probability inequalities (Tchebyshef, Markov, Jensen). Modes of convergence, weak and strong laws of large numbers, Central Limit theorems .

Markov chains with finite and countable state space, classification of states, limiting behaviour of n-step transition probabilities, stationary distribution, Poisson and birth-and-death processes.

Standard discrete and continuous univariate distributions. sampling distributions, standard errors and asymptotic distributions, distribution of order statistics and range.

Methods of estimation, properties of estimators, confidence intervals. Tests of hypotheses: most powerful and uniformly most powerful tests, likelihood ratio tests. Analysis of discrete data and chi-square test of goodness of fit. Large sample tests. Simple non-parametric tests for one and two sample problems, rank correlation and test for independence. Elementary Bayesian inference.

Gauss-Markov models, estimability of parameters, best linear unbiased estimators, confidence intervals, tests for linear hypotheses. Analysis of variance and covariance. Fixed, random and mixed effects models. Simple and multiple linear regression. Elementary regression diagnostics. Logistic regression.
Multivariate normal distribution, Wishart distribution and their properties. Distribution of quadratic forms. Inference for parameters, partial and multiple correlation coefficients and related tests. Data reduction

Techniques: Principle component analysis, Discriminant analysis, Cluster analysis, Canonical correlation.

Simple random sampling, stratified sampling and systematic sampling. Probability proportional to size sampling Ratio and regression methods.

Completely randomized designs, randomized block designs and Latin-square designs. Connectedness and orthogonality of block designs, BIBD. 2K factorial experiments: confounding and construction.

Series and parallel systems, hazard function and failure rates, censoring and life testing,

Linear Programming Problem:

Introduction, definition of related terminology such as constraints, objectives function, optimization, different types of linear programming (L.P.) problems, mathematical formulation of L.P problems, graphical method of solution for problems in two variables, feasible and infeasible regions, feasible and infeasible solutions, optimal feasible solutions (up to three non-trivial constrains)

Simplex methods, duality. Elementary queuing and inventory models. Steady-state solutions of Markovian queuing models: M/M/1, M/M/1 with limited waiting space, M/M/C, M/M/C with limited waiting space, M/G/1.